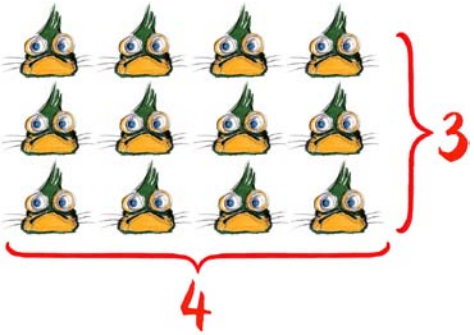
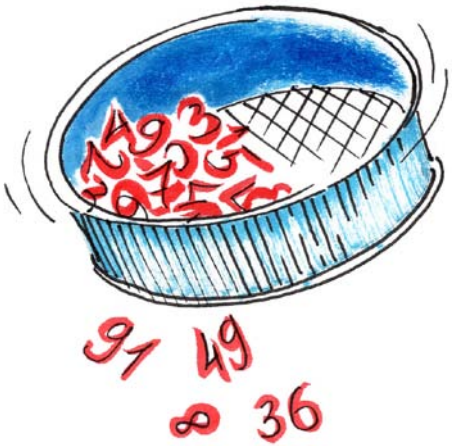


Prime and Composite Numbers



Karina Kudymova, 2013

If you have 10 coins can you arrange them into a rectangle?

Sure, that's easy

What about 11 coins?

Hmm, that doesn't seem to work...

If the coins can be laid out in a rectangle then their number is a product of two factors.

$21 = 3 \times 7$

$2117 = 73 \times 29$

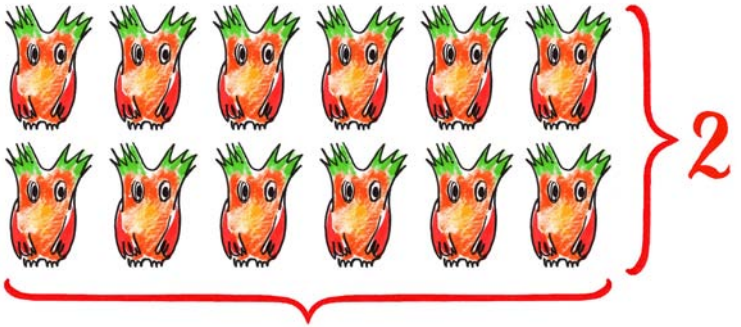
$81 = 9 \times 9$

$413 = 59 \times 7$

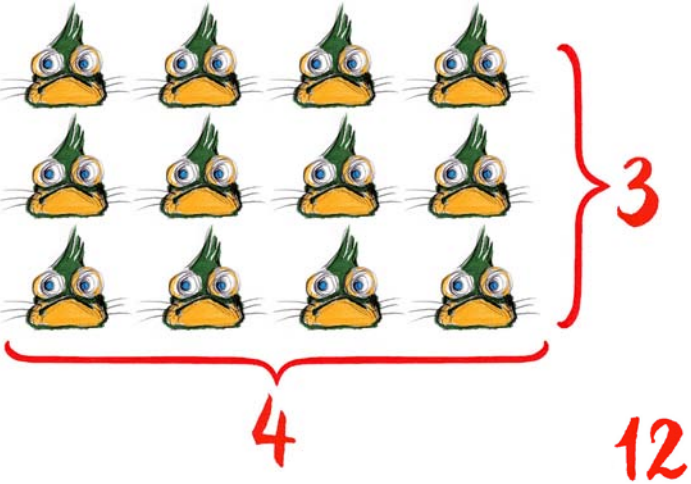
$10 = 2 \times 5$

We call such numbers *composite*.

Some numbers can be written as different products...



$12 = 2 \times 6$



$12 = 3 \times 4$

But it doesn't matter, they are still composite.

Numbers that cannot be written as a product are called *prime*.

A careful reader might note that any number can be written as a product of itself and **1**, but that doesn't count.

7 19 541
47 311 811

Also, number **1** is considered neither prime nor composite.

Prime numbers can be very large, for example this number is a prime:
1,363,005,552,434,666,078,217,421,284,621,279,933,627,102,780,881,053,358,473

The largest known prime has 12978189 digits!

If each digit is one inch wide this number will stretch from Washington, DC to New York City!

Can you just look at a number and tell if it's a prime or a composite?

You often can...

Is number 268,435,454 a prime?

It's a big number but it is *even* so it is divisible by 2 and it must be composite

$$268{,}435{,}454 = 2 \times 134{,}217{,}727$$

What about 268,435,455?

This number ends with 5 so it must be divisible by 5 and is also composite

$$268{,}435{,}455 = 5 \times 53{,}687{,}091$$

To prove that a number is composite we need to show that is divisible by a smaller number.
Let's review simple divisibility rules.

	Examples		
A number is divisible by 2 if its last digit is divisible by 2 (it can be 0, 2, 4, 6, or 8)	35,678	✓	last digit 8
	35,679	✗	last digit 9
A number is divisible by 3 if the sum of its digits is divisible by 3	2,412	✓	sum of digits 9
	2,413	✗	sum of digits 10
A number is divisible by 5 if its last digit is either 0 or 5	73,455	✓	last digit 5
	73,456	✗	last digit 6

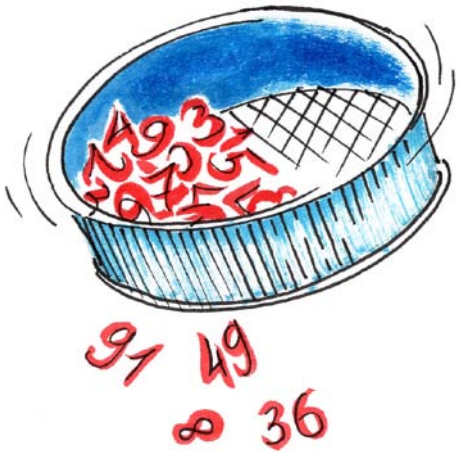
To find primes we don't need to check for divisibility by composite numbers and should only test for prime factors. For example, if we know that a number divisible by 2 or 3 we don't need to check for divisibility by 6.

Let's find all the primes that are less than 100

Listing all 100 numbers we can cross out the ones divisible by 2, 3, 5, and 7.

Can you tell why we don't need to test larger factors?

The remaining numbers are primes



This is an ancient algorithm called the Sieve of Eratosthenes

1	**2**	**3**	4	**5**	6
7	8	9	10	**11**	12
13	14	15	16	**17**	18
19	20	21	22	**23**	24
25	26	27	28	**29**	30
31	32	33	34	35	36
37	38	39	40	**41**	42
43	44	45	46	**47**	48
49	50	51	52	**53**	54
55	56	57	58	**59**	60
61	62	63	64	65	66
67	68	69	70	**71**	72
73	74	75	76	77	78
79	80	81	82	**83**	84
85	86	87	88	**89**	90
91	92	93	94	95	96
97	98	99	100	101	102

Another ancient result is that there are infinitely many primes. The sequence 2, 3, 5, 7, 11, 13, 17, ... never ends!

This theorem was proven by Euclid around 300 BC

To prove it, imagine that there were only finite number of primes. Then we could pick the largest of them. Now think of the product of all numbers up to the that largest prime. The number right after it cannot be prime because it's bigger than what we assumed was the largest prime. But it cannot be divisible by any number less than our largest prime so if it is composite then its prime factors have to exceed the largest prime number.

Are prime numbers useful? Yes! For example, computers often rely on them for cryptography needed for secure information interchange.