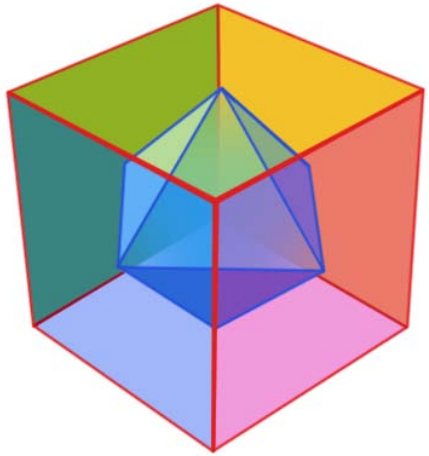
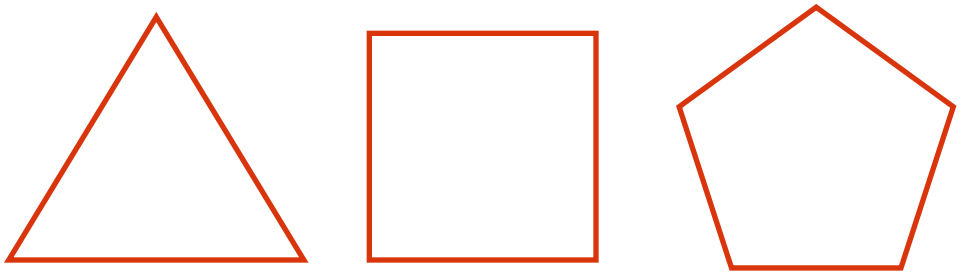
Regular
Polygons and Polyhedrons



Karina Kudymova, 2013

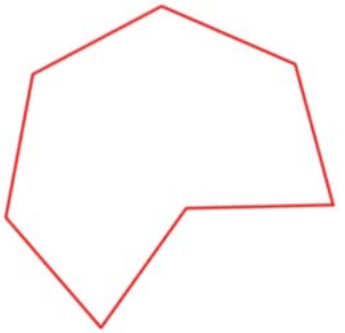
Regular polygons and polyhedrons

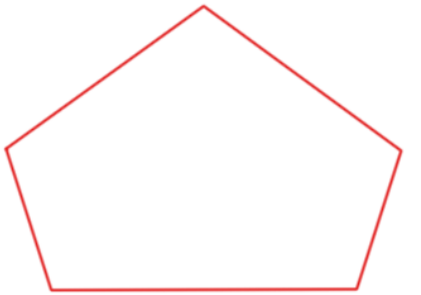
What do these triangle, square and pentagon have in common?



They are *regular polygons*.

Regular polygons have equal angles and equal sides.



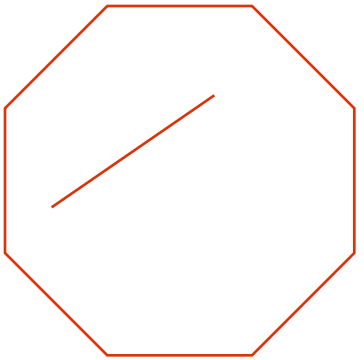


If only the sides are equal the polygon might be irregular.

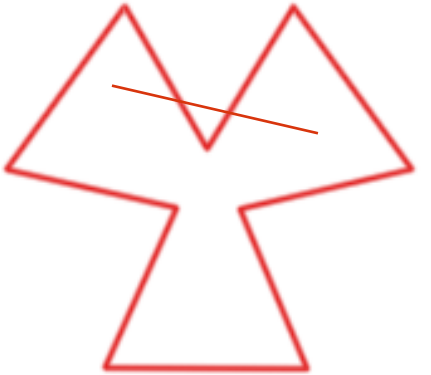
Similarly, if only the angles are equal the polygon might be irregular.

Regular polygons are always convex

An object is convex if a line segment connecting any two points in it is fully inside the object

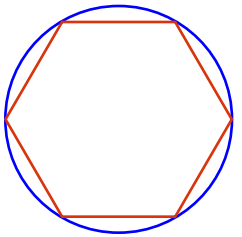


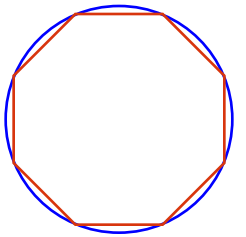
Convex

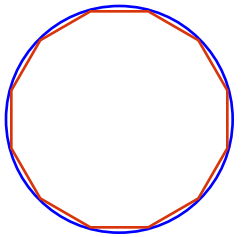


Not convex

All vertices of a regular polygon lie on a common circle.







And as the number of sides increases regular polygons get closer to a circle.

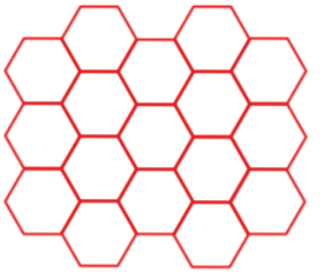
Which regular polygons can cover a plane without any gaps? This is also called regular tiling.

We can use triangle or square floor tiles.

Can pentagons cover a plane without gaps?

No, at most three pentagons can share each vertex of a tiling. But that leaves some gaps unfilled.

Bees use hexagons to build honeycombs.



And if regular polygons have more than six sides, we cannot even fit three of them on a plane to share a common vertex.

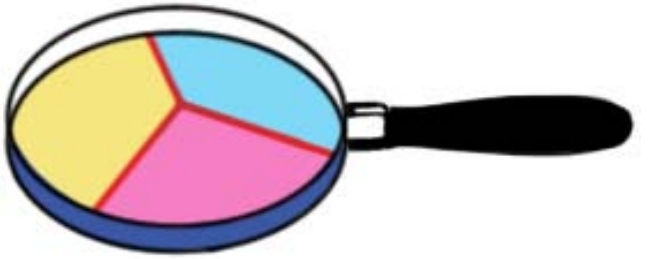
Polygons exist on a 2D plane, but in 3D space we have polyhedrons. A polyhedron is regular if:

- All of its faces are congruent regular polygons
- It is confex
- Same number of faces meet at each vertex

Faces is just another word for *sides*.

Congruent is just a fancy way of saying *same shape and size*

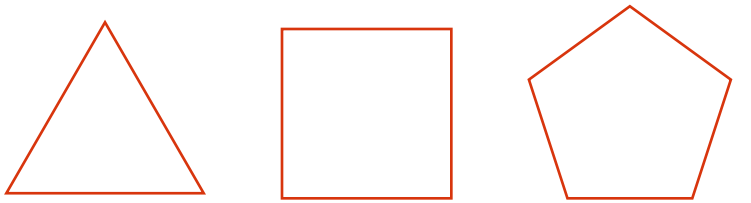
Also, another fancy way to say *polyhedrons* is *polyhedra*.



At least three faces can meet at each vertex.

Let's count all regular polyhedrons and prove that we didn't miss any.

Which regular *polygons* can be faces of a regular *polyhedron*?

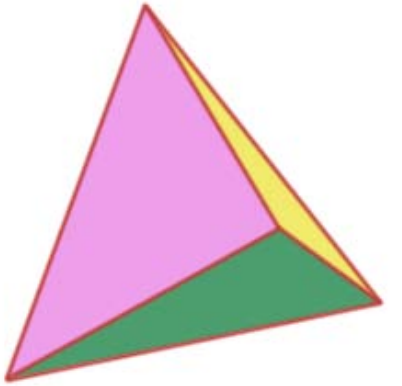


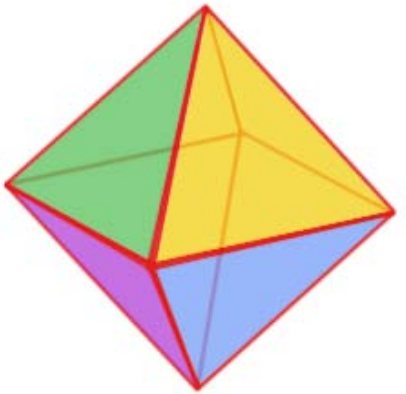
A triangle, a square and a pentagon

Regular polygons with six or more sides cannot form a vertex with three faces

Let's start with triangles.

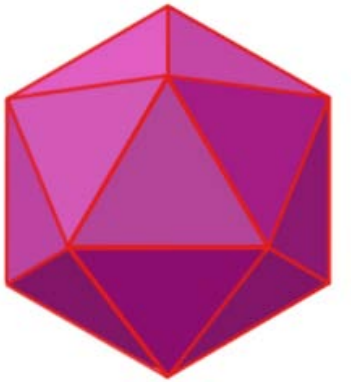
If only three triangles meet at each vertex we get a *pyramid* or *tetrahedron*





What if we use four triangles instead? Then we get a diamond-shaped polygon called *octahedron*.

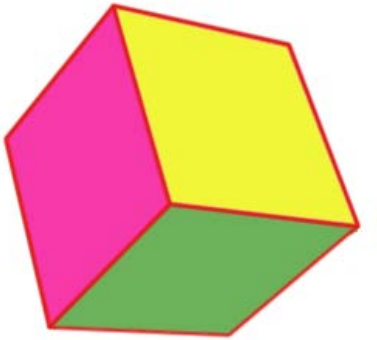
If five triangles meet at each vertex we can construct an object called *icosahedron*.
It has 20 faces (*icosa* is ancient greek for 20).



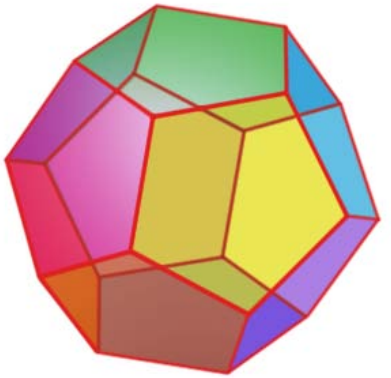
We cannot put six or more equilateral triangles at each vertex of a polyhedron.
Can you tell why?

We just constructed three regular polyhedrons with triangular faces. Let's move on to squares. Can we make a regular polyhedron with square faces?

Of course, a cube (also known as a *hexahedron*)



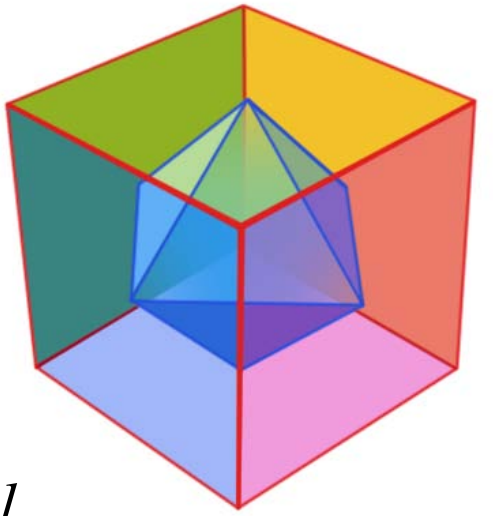
Are there any other regular polyhedrons with square face?
No: we need at least three squares for each vertex, but putting four squares will make it a plane, not a polyhedron.



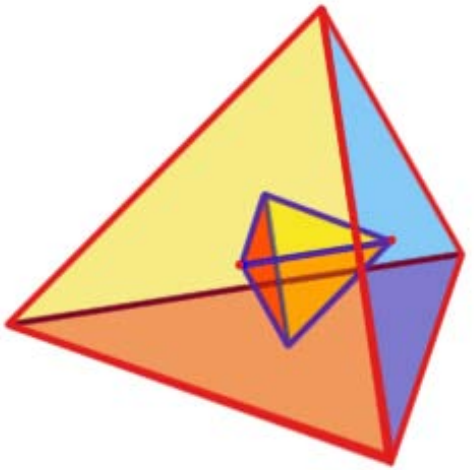
Finally, the only regular polygon with pentagon faces is a *dodecahedron*.
It has 12 faces.

The cube has 6 faces and 8 vertices and the octahedron has 8 faces and 6 vertices.

In fact, if we take one of these two polyhedrons, mark centers of each face and create a polyhedron with corresponding vertices, we'll get the other one.



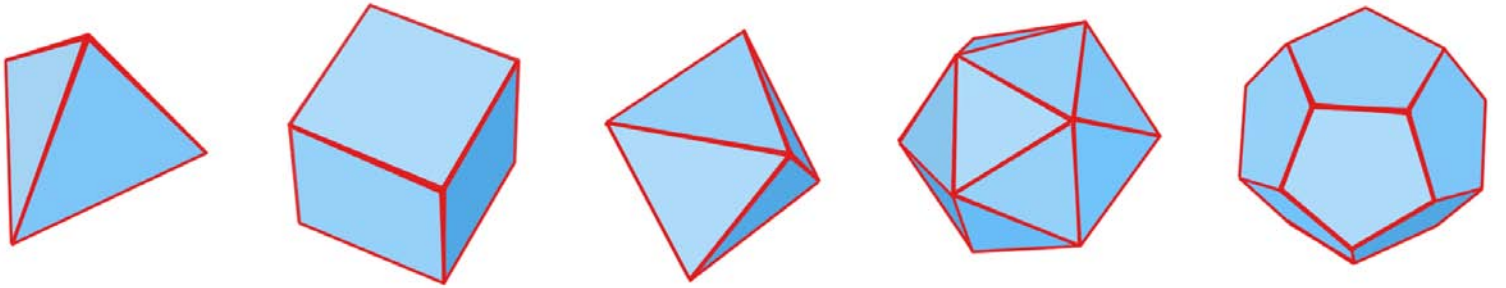
We call such two polyhedrons *dual*.



Cube and octahedron are dual and so are dodecahedron and icosahedron (their numbers of faces and vertices are 12 and 20).

Also, a tetrahedron is a dual of itself.

Five regular polygons that are called *Platonic solids*.



They are named after an ancient greek philosopher Plato who looked for philosophical meaning in polyhedrons.