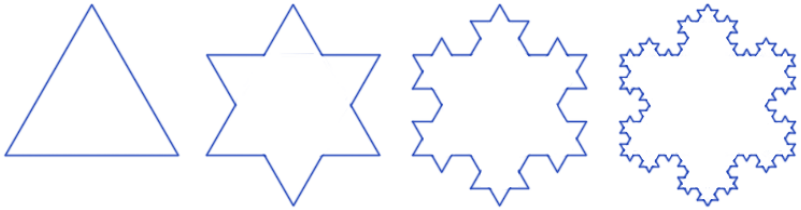
Fractals



Karina Kudymova, 2013

How can we measure the length of Lake Tahoe shore line?

Well, we could drive on the road that goes around the lake and measure the distance.

But the real shore line has many more curves than we followed and is actually longer.

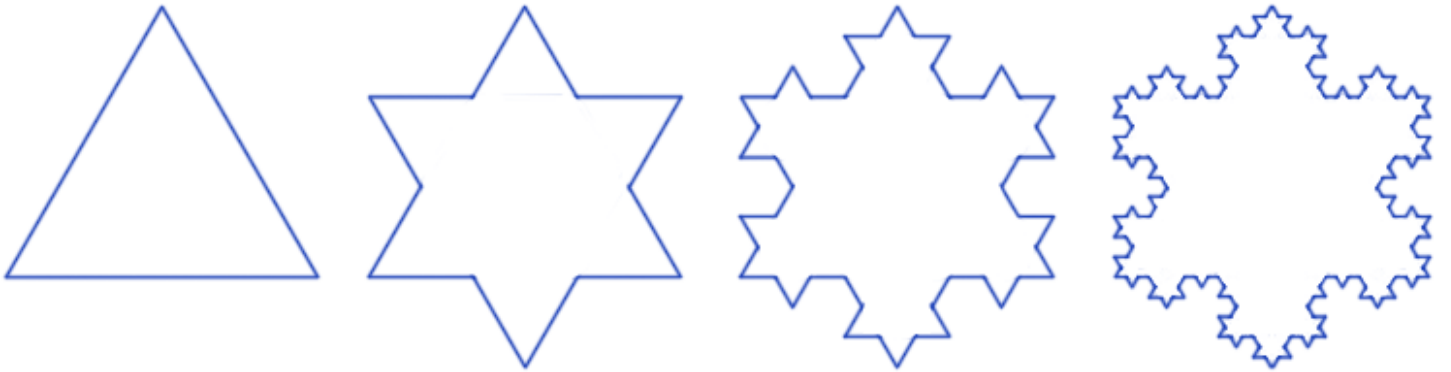
If we try to follow the curves of the shore exactly, there will definitely be details that we won't be able to measure.

The lake's shore is an example of a shape with parts that look the same but are different sizes. These shapes are called *fractals*.

Here are some other examples of how natural shapes can be fractals:

In math, fractals are usually put together by repeatedly making similar changes in a pattern that changes its size.

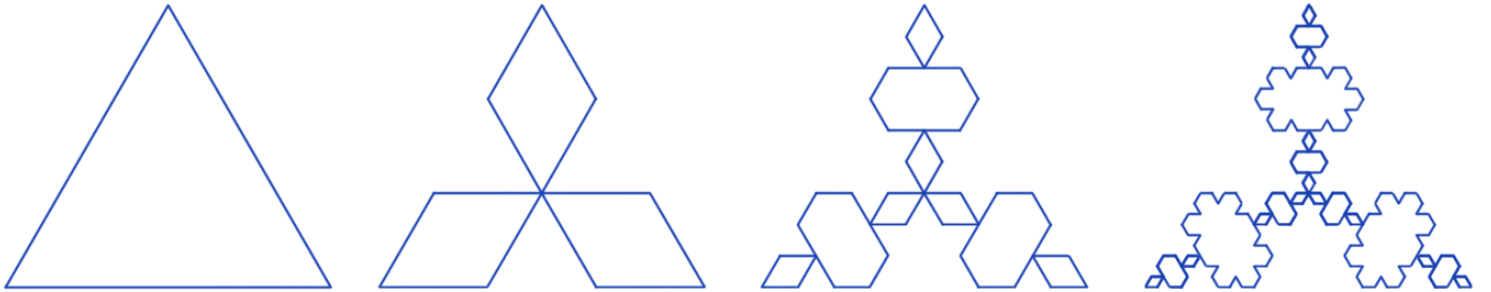
Here's an example. This is called the *Koch Snowflake*.



We start with one triangle. Next, we add smaller triangles on each side. Then we add even smaller triangles and just keep going.

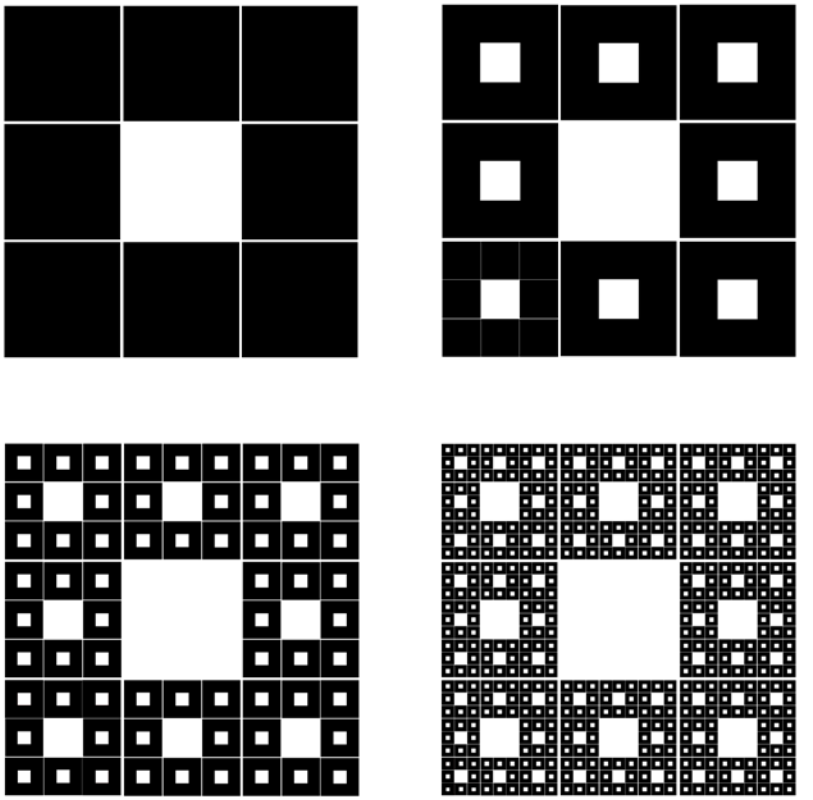
Do you see how the snowflake becomes more and more complex?

What if we took the same triangle, but instead of adding them on the outside, we put them on the inside? Then we would make what's called an *anti-snowflake*.



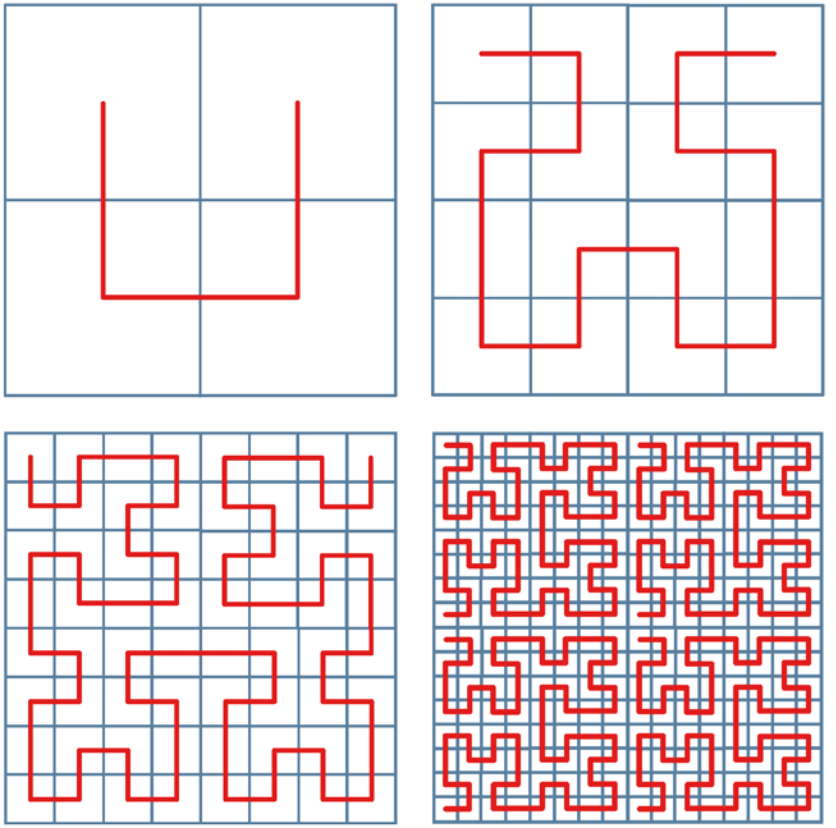
The anti-snowflake fits nicely with the Koch snowflake. Something that's interesting about both snowflakes is that every time we change it, its perimeter increases by a factor of 4/3. If we never stop adding on triangles, the perimeter of the snowflake becomes infinite.

Another example of a fractal is the *Serpinski carpet*. We take a square, divide it into nine 3x3 tiles and remove the middle tile. Next, we take each of the 8 tiles that are left and divide them into 9 even smaller tiles. After that, we take out the tile in the middle and keep going.



See how the carpet becomes more and more complex? Can you think of another carpet like this but with triangles instead of squares?

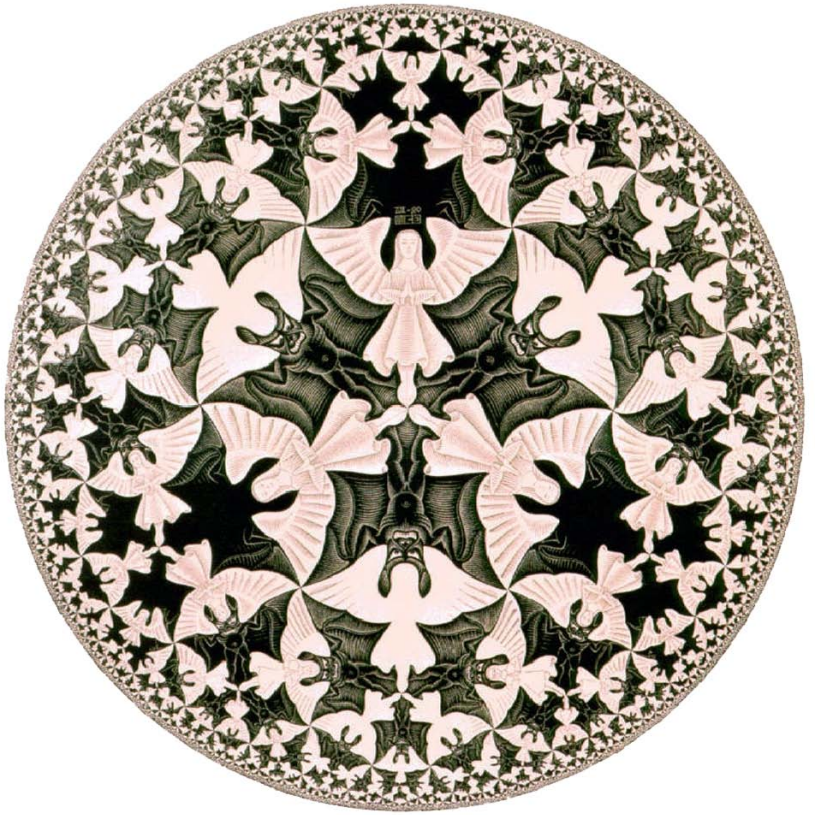
In the example from before, we started with a flat (2D) object and cut out pieces. Then we ended up with a shape that's is so full of holes it has zero area. We can go in the opposite direction too. We can make a 1D curve so complex that it fills an entire 2D region.



This is an example of *Hilbert space-filling curve*. Can you see how it is constructed?

The Serpinski carpet and the space-filling curve are neither 1D nor 2D objects. They have fractional dimensions and that's why they are called *fractals*.

A famous Dutch artist M.C. Escher often drew inspiration from fractals in his work



Angels and Demons

Circle Limit 5

Fractals are also used in computer graphics and animation to depict complex shapes, such as realistic landscapes.