

Coins and Weighings



In this section you'll be required to find a counterfeit coin that looks exactly like the genuine ones but has different weight. To compare weights you'll use a pair of scales balance that can only show if two sides have equal weight or if one side is heavier.



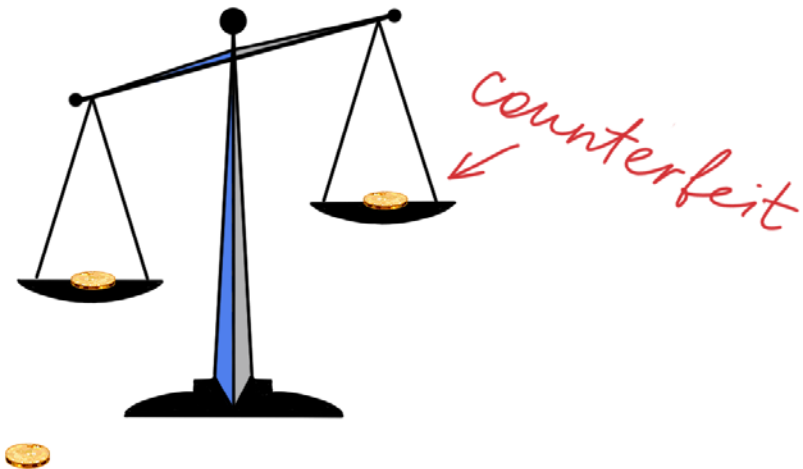
If the coin you are looking for is lighter than the others, can we find it in *one* weighing?



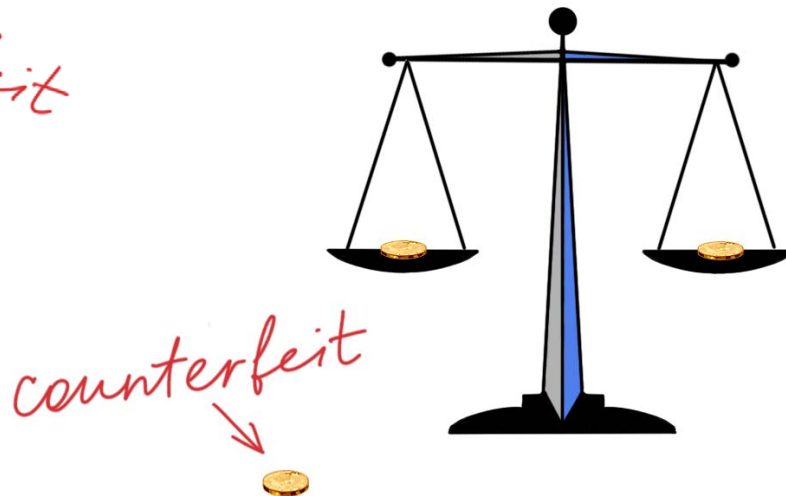
Well, if we have just two coins, it's easy: compare them and pick the lighter one

But what if we have three coins and one of them is lighter?
Can we still find it with one weighing?

Sure, compare any two coins,
if one is lighter - that's the one
we were looking for.



If the weight of the two
coins is equal, the remaining
one must be counterfeit.



Now we have nine coins and exactly one of them is lighter than the others. Can we find it with just *two* weighings?

Let's arrange nine coins in triplets and weigh two of them.



Like in the previous problem, if one side is lighter, the counterfeit coin is there and if the sides are equal, the coin is in the group that is not on the balance.



Either way, after one weighing we know the group of three coins that has the counterfeit one and in one extra weighing we can find it.



How would you find the lighter coin in a group of 27 if you have three weighings?

Let's change the problem slightly. Now we have five coins and exactly one of them is counterfeit, but we don't know if it's heavier or lighter. Can we find it in two weighings if we also have a few coins that we know are genuine?

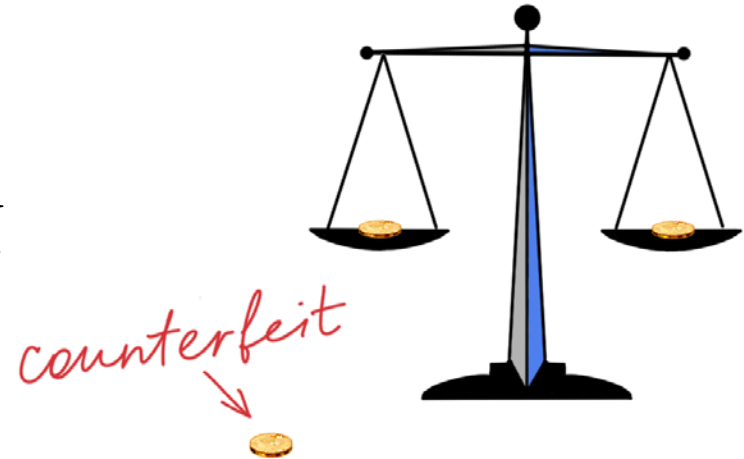
Let's take any three coins from our group of five and compare them against three genuine coins.



If the weights are equal, the counterfeit coin is one of the remaining two. Just compare one of them against a genuine coin, if the weights are different, that's the one, if not, then the last one is counterfeit.

But if three selected coins weigh differently than the genuine coins, then the counterfeit coin must be one of them. We'll also know if the coin is heavier or lighter than the rest depending on this weighing.

In the next weighing we'll compare any two out of these three coins and if the weights are the same, then the last coin is the culprit.



If the weights are different, then one of these two is counterfeit, but which one? Well, if the first three tested coins were heavier than the genuine ones, then the heavier of the two is counterfeit. If they were lighter, then it's the other one.



Suppose we have 13 coins. One of them is counterfeit and has a different weight from the others. Can we find it in three weighings?

Let's start with comparing four coins on each side. If their weight is equal, the counterfeit coin must be in the remaining 5 and we know how to find it with two remaining weighings.



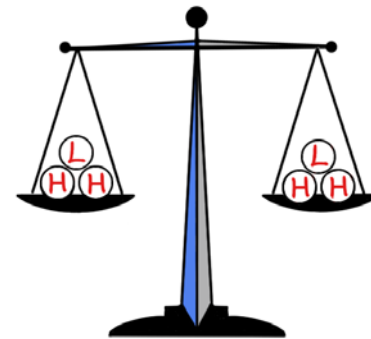
There is a counterfeit coin here



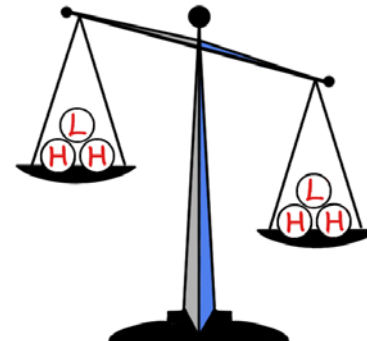
genuine

If the weights are different, we'll mark the 4 coins on the heavy side of the balance with "H" and the other 4 with "L".

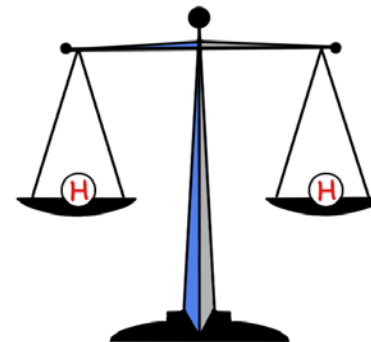
Let's put two "H" coins and one "L" coin on each side and compare. If they are equal, the coin we are looking for is among two remaining "L" ones, and we can easily find it with one weighing.



If they are different, we can eliminate two "H" coins that are on the lighter side and one "L" coin on the heavy side.



If we compare two remaining "H" coins and one is heavier, that's the one we are looking for.



If they are equal, the remaining "L" coin is the one.

Now we'll consider a different problem. The mint stamps out genuine coins that weigh 10 grams, but by mistake they made a sack of coins that weigh just 9 grams.

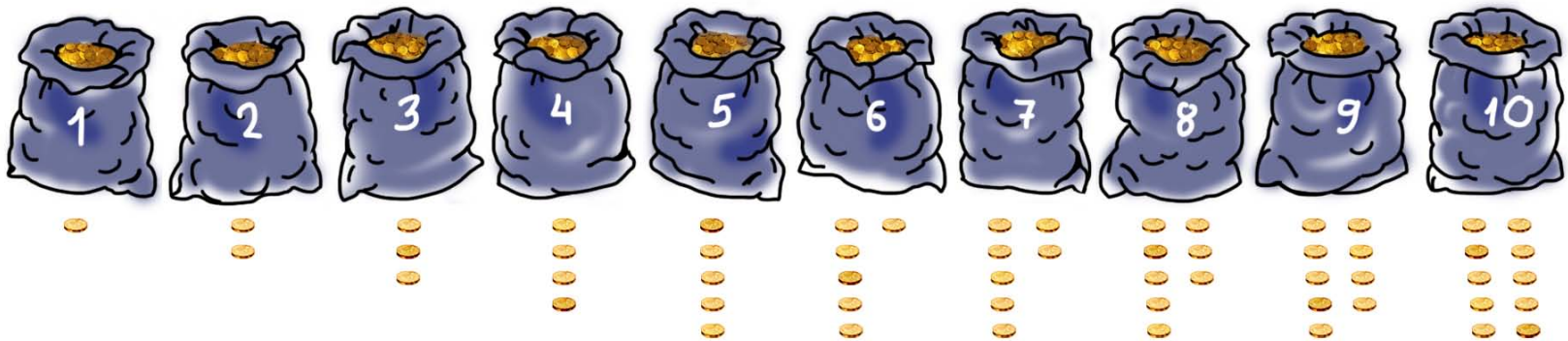


There are 10 sacks total and only one of them has all coins minted by mistake.

You have a scale that shows the exact weight in grams of all the coins that you put on it. Can you find out which sack has 9 gram coins with just *one* weighing?



It turns out you can. Take one coin from the first sack, two from the second, three from the third, and so on. If all coins weighed 10 grams, the total weight would be 55 grams (can you tell why?).



But because some coins weigh 9 grams, the result will be smaller. If it's smaller by 1 gram, it must be the first sack, if it's smaller by 7 gram, the bad coins are in sack number 7 .